JUGOSLAV INSTITUTE OF ECONOMIC RESEARCH

THE OPTIMUM RATE OF INVESTMENT

Branko Horvat

REPRINT № 3
FROM *THE ECONOMIC JOURNAL*
DECEMBER 1958

THE OPTIMUM RATE OF INVESTMENT[1]

Introduction

1. The problem discussed in this paper is generally known as the problem of the " optimum rate of saving." The change in the terminology suggested in the title is not only a formal matter. It is intended to indicate the difference in the approach.

The discussion of the optimum rate of saving has hitherto been based on various considerations concerning utility maximisation. Essentially they involved comparisons between " present sacrifices " and " future gains." The first solution offered was that by F. Ramsey[2] as early as 1928. Ramsey's method consisted in comparisons between potential satisfactions in the state of " Bliss " (the maximum obtainable rate of utility) and the satisfactions of the current income. The most recent solution comes from Professor Tinbergen,[3] who derives it from particular assumptions about the shape of utility functions (logarithmic and hyperbolic) and the value of the " psychological discount " (the rate of time preference).

Both Ramsey's and Tinbergen's approach, as well as other attempts along similar lines, yield determinate solutions. But they are obviously not designed for being put to practical use: the concepts used exclude statistical measurement. Therefore in practical work another approach is often used. In planning economic development usually some constraints are postulated, *e.g.*, consumption must not fall below certain level; they are arbitrarily defined and then, considering these constraints as data, the saving decision is reached.

Thus so far we have been left with a choice of two alternative solutions: one determinate but impossible and the other practically possible but theoretically indeterminate. It is the scope of this paper to examine a possibility of a third solution, which would be both practicable and determinate.

2. For the start let us define a few basic concepts which will be used throughout the analysis.

(1) Gross social product and, consequently, the additions to gross social product will be conceived and computed net of services not intimately connected with material production (public administration, defence expenditures, etc.; transport and trade services are included). This definition of *GSP* is adopted partly with regard to available empirical data and partly because it simplifies the analysis.

[1] The basic ideas elaborated in this paper were first expounded in a talk given to the Staff Seminar in Economics at Manchester University in May 1957. I benefited from the discussion at the Seminar and afterwards. Professor Devons and Dr. Martin have been kind enough to read the manuscript and discuss it with me. But, of course, I am alone responsible for it.

[2] F. P. Ramsey, " A Mathematical Theory of Saving," Economic Journal, 1928.

[3] J. Tinbergen, " The Optimum Rate of Saving," Economic Journal, 1956.

(2) The phrase "productive investment" will mean investment producing an increment in *GSP* as defined in the preceding paragraph.

(3) In addition to the general distinction between investment and consumption, two further distinctions will be made. Thus social product will be considered to consist of four different parts. They are:

(*a*) Investment in the material factor of growth which causes the expansion of productive fixed and circulating assets. This part of *GSP* will be referred to as "investment."

(*b*) Investment in the human factor of growth (termed as *A*-factor) which increases the ability of the society to produce material goods. To distinguish (*b*) from (*a*) it may be called "productive expenditures."

(*c*) and (*d*) The remainder of social product is divided between personal and communal consumption. As a matter of fact, parts of (*c*) and (*d*) are physically identical with (*b*). The reason for putting certain consumption goods and services into (*b*) is that their consumption leads to an increase of *GSP*. If these consumption goods and services lose their positive productive effect, they will be classified only as (*c*) and (*d*). The (*c*) and (*d*) parts of *GSP* will be referred to as "consumption."

(4) It will be assumed that maximisation of consumption, as just defined, is the sole aim of the society's productive efforts. This will imply that if there is a choice between smaller and greater consumption, other things being equal, the society will choose the alternative with greater consumption. If other things are not equal, the society will face a choice—*e.g.*, maximum consumption as against maximum military strength—which falls outside this paper and, indeed, outside the scope of economics. However, society will normally not have to choose between greater consumption and greater production. One of the main purposes of this paper is to prove the identity of these two aims in the real world.

The Concept of the Absorptive Capacity of the Economy

3. Instead of maximisation of utility, it is proposed to consider the maximisation of production through time. Production is maximised with respect to a specified period, the length of which is determined by some physical properties of the economic process.

Maximisation involves not only the allocation of factors of production now, but also the adjustment of their various rates of expansion in the future. The potential effect of the optimum adjustment of the growth rates of factors is defined as the absorptive capacity of the economy. The easiest way to use this concept is to conceive the economy as a giant productive capacity capable of being expanded at a certain *maximum* rate, also at a lower rate, but *not at a higher rate*. Any additional inputs (investment) would not produce *additions* to but *reductions* of output. Or, applying (with caution) the conventional terminology, marginal efficiency of investment will be zero or negative.

Zero marginal efficiency of investment does not necessarily mean zero marginal efficiency of capital.[1] The latter is a static while the former is a dynamic concept. It means even less that the rate of interest is zero. In fact, the belief that the positive rate of interest is an evident proof that the marginal efficiency of investment is still positive—seems to have prevented a further exploration of the practicability of production maximisation. It has been observed that in normal circumstances the rate of interest has never vanished; it existed with investment expenditures however large. It was therefore believed that only an extraordinarily high rate of investment would drive the marginal efficiency of investment towards zero. And an extraordinarily high level of investment would imply an extraordinarily low level of consumption. One has therefore to weigh "present sacrifices" with "future gains," and this is the domain of the utility calculus.

However, the actual economic process is somewhat different. To show this in the simplest possible way let us consider its characteristics in a planned economy.

Investment–Production Function

4. Investment is made to increase production. There is therefore some relation between the addition to the output stream and the productive investment which has caused it. There is also a time lag between the cause and the effect known as the maturation period of investment (m). The ratio between additional output[2] in this year and the investment[2] made m years earlier we shall term "production coefficient" $\left(p = \frac{\Delta GP_t}{I_{t-m}}\right)$. The change of p as a function of investment will be called "investment–production function" (*IP*). This function may be diagrammatically represented in the following way:

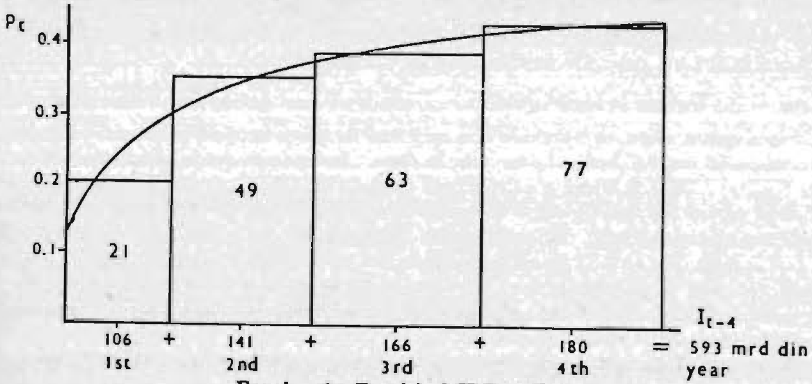

Fig. 1.—An Empirical *IP* Function

[1] This has been pointed out by A. Lerner, who coined his famous symbol *mei* (*Economics of Control* (New York, 1944), pp. 260 ff.).

[2] Output is defined as gross product and investment as the addition to the stock of productive capital, *i.e.*, statistically as gross investment net of replacement. However, in the following empirical illustrations gross investment figures will be used because of the lack of reliable data for net investment.

It is fortunate that here we are able to make use of an empirical diagram.[1] The need for representing three variables in a two-dimensional co-ordinate system has made for a little unusual construction of the diagram; it is, however, perfectly simple once the meaning of it is grasped. The ordinate shows the values of the production coefficient, which in four (adjusted) years has risen from 0·2 in the first year to 0·43 in the last one. The abscissa shows successive doses of investment expressed in milliards of dinars. The additions to social product are related to investments made four years earlier (maturation period). In each successive year the "dose" of investment has been increased, bringing the total for the whole period equal to 593 milliard dinars. If we multiply "investment doses," represented by the segments of the abscissa, by the respective production coefficients, we get increments in output which, in the diagram, are represented by the areas of the respective columns. The figures inside the columns indicate the size of these increments. The total increase in output in the period under consideration is equal to the sum of the areas of the four columns, $\Delta GP = \sum_{1}^{4}\Delta GP_t = 210$ milliard dinars. It is this last figure which will become our main object of analysis as we proceed. Finally, the free-hand drawn curve is intended to approximate continuous changes in the production coefficient. The area under the curve is, of course, equal to the sum of the increments in output, $\int_0^{593} p(I)dI = \sum_{1}^{4}\Delta GP_t$.

The main lesson this diagram teaches us consists in the demonstration of the possibility of great changes of production coefficients. In four years the value of p has increased by more than 100%. This is obviously not due to the technological changes only.[2] Thus we have to investigate the characteristics of our *IP*-function more carefully.

[1] The diagram refers to Yugoslav economy and is taken from an unpublished study by the author prepared for the Federal Planning Bureau. Investment (gross) and production (gross) figures (prices: 1952) are four-year averages (period 1947–57), because it was found that the maturation period for the Yugoslav economy is about four years. The data refer to industry (manufacturing and mining). The production coefficient for the whole economy increased in the same period from 0·18 to 0·37 (gross investment does not include investment into houses, schools, hospitals, etc., and gross product does not contain the value of services). Industrial rather than whole economy *IP*-function is shown because the period is too short to eliminate the influences of the agricultural weather cycle from the latter.

[2] The causes of this spectacular improvement may be tentatively described as follows:

(1) The most important impulse was given by the far-reaching economic reorganisation started in 1951–52. The managerial system with the rigid administrative central planning and control (more or less a copy of the Soviet economic organisation) has been gradually replaced by a combination of workers' management plus global planning through market instruments plus some central decisions concerning major investment projects.

(2) The tempo of investing has been slowed down and the know-how has been gradually improving.

(3) Changes in the structure of investment occurred: after the basic industrialisation programme had been accomplished by the first Five Year Plan (1947–52) relatively more was invested in extensions and relatively less into new factories. A survey made recently in Croatia showed that the capital productivity of extensions was about 50% higher.

5. The assumption from which we started was that the absorptive capacity of the economy was limited: additional investment beyond certain limits brought about negative increments in output. Thus we may say that *IP*-function depends on the quantity of investment (I) and on the absorptive capacity of the economy (A) and on the speed of their expansion $\left(\frac{dI}{dt}, \frac{dA}{dt}\right)$, or in symbols

$$p = f\left(I, \frac{dI}{dt};\ A, \frac{dA}{dt}\right)$$

Investment has to be absorbed by the economy. How well this will be done depends on the human factor A. The complex factor A may, perhaps, be made a function of four basic factors (policy variables) and of their changes: of personal consumption, health, knowledge, and of economic and political organisation. In addition, all other relevant characteristics of the economy may be conveniently lumped together as a single exogenous factor E.[1] Thus we get:

$$\left(A, \frac{dA}{dt}\right) = g\left(C, \frac{dC}{dt};\ H, \frac{dH}{dt};\ Kn, \frac{dKn}{dt};\ O, \frac{dO}{dt};\ E, \frac{dE}{dt}\right)$$

The factors (apart from E) are arranged according to the degree of their tangibility and physiological importance for human beings; their importance for the economic process (measured by changes which various factors may provoke) is roughly of the reverse order. The meaning of A-factors may be described very briefly in the following way.

Personal consumption has a great incentive value in a poor society. It may, therefore, influence the productivity appreciably. It may be added that in a growing economy the alternative to be chosen normally lies in the direction of an increase; the possibility of a decrease of consumption cannot normally arise in a rationally constructed scheme of economic development as will be shown presently. (The only two probable exceptions seem to appear in societies with an extreme inequality in the distribution of income or where mass enthusiasm had been created; in both cases total consumption may temporarily stagnate or even decline without causing a contraction of A.)

That the improvement in the *health* standard leads to an increase in the productivity of labour is notorious. A suitable generalisation will perhaps be provided by quoting the estimate of J. J. Spengler that the potential productivity of population of underdeveloped countries would rise 20–30%

[1] The exogenous factor E has been inserted to make the algebraic presentation formally correct. As, by definition, E is not a policy variable, we shall not be concerned with it, and we shall always assume that E is somehow given and known. In actual planning, of course, the analysis of E is of an extreme importance.

if the age composition and state of health of their people could be "Westernised." [1]

Factor *knowledge* comprises all degrees of skill, including scientific research. The experience of planning seems to suggest that knowledge (and certainly not investment resources [2]) is the most important scarce factor in underdeveloped countries with otherwise favourable social climate. Thus growth of "know-how" is likely to pose the limits to the general economic development.

The last policy factor, termed *economic and political organisation*, refers to the institutional set-up of the economy. Inadequate economic organisation and political discontent and instability are likely to upset the productive effect of all other factors and so to minimise the value of A. But factor O may also considerably increase the value of A, as may have happened, for instance, in India as a result of having achieved political independence or in China after social revolution. Factor O is usually taken as a datum in the economic analysis, and nothing prevents us from following this traditional line here too. Yet there is also some justification in assuming that in a rationally organised society the shaping of such an important productive factor will not be left to the haphazard play of blind social forces, but will be undertaken with at least some degree of rationality as it is the case with factors C, H and Kn. Planned economy may be assumed to experiment not only with micro- but with macro-organisation as well. And the effects are not as intangible and immeasurable as they may seem at first glance. It suffices to compare the economic results of various solutions applied in similar situations. For instance, the agricultural policy of Poland with that of Eastern Germany, or the economic results of the managerial system in Bulgaria and in Soviet Union with those of workers' management system in Yugoslavia, or underdeveloped countries with planned and unplanned economies. In any case, given or changeable, O is an important limitational factor.

6. The foregoing considerations show that A is in principle measurable and that in practice its value can be well approximated. The next step in the analysis consists in an examination of the relation between A and I. An image of a vessel (A) into which liquid (I) is poured from outside springs to mind. However, the picture is a little more complex than that. The

[1] J. J. Spengler, "The Population Obstacle to Economic Betterment," *American Economic Review, Proceedings*, May 1951, p. 344. Spengler also compares the estimate of D. Ghosh for India, where about 22½% of the national income is spent on maintaining those who die before they reach the age of 15, with that for England, where the corresponding figure is only 6½% (*ibid.*, p. 351).

[2] This is also obvious from the structure of *IP*-function. Investment is something relative, it is the share of investment in social product which is analytically meaningful. Given enough time, the share of investment may be increased as far as we like (short of 100%). The question is only whether the economy will absorb it. We are accustomed to think of investment as an independent factor of growth. However, in a rationally organised economy it would be more appropriate to treat it as a product of growth. Given A, the economy is able to produce any amount of I which can be productively applied.

vessel itself produces the liquid, which in turn causes the expansion of the vessel. This feedback operates with diminishing force, and the movement comes to an end when the vessel is filled up and additionally generated liquid is uselessly spilt. This is the point of optimum.

In other words, A and I are obviously interrelated. Investment makes possible the expansion of the community's absorptive capacity; A imposes the limits to the productive application of I. The absorptive capacity may now be redefined to mean the ability of individuals and of the society as a whole to manipulate the stream of output increments. This ability is limited because there is a given level and a limited speed of potential expansion of the will to work, of the state of health, of the number of skilled workers and scientists and of the institutional readjustments. Being limited, A may be taken as a datum (although a changing one) and I may be defined as its function, $I = f(A)$. So defined, investment is the *maximum investment which may be productively applied* in a given economy. Thus the *optimum* path of the investment–production function will be given by

$$p_{\text{opt}} = F[I(A)]$$

In this way the optimum rate of investment is determined.

Maximisation of Output Increment

7. For the purpose of application it will be necessary to reformulate slightly the last result using the inverse functional relationship of I and A. The problem will now be formulated as follows: try various quantities, collections and sequences of I, and for each of them adjust A in the optimum way. Then A becomes a function of I, $A = \phi(I)$, and various possible IP functions will be expressed by $p = \Phi[A(I)]$. Our task is now to integrate all these functions and to find the maximum integral

$$\text{Max. } \Delta GP = \text{Max.} \int_0^{I_m} \Phi(I)\, dI$$

Maximum integral represents the maximum possible increase of production in a specified period of the length m. As any other combination of investment would result in a smaller addition to output, this integral provides the formula for the optimum rate of investment.

There is also a third alternative method of finding the optimum rate of investment. Define capital coefficient as $k = \frac{1}{p}$. Increment of output in a specified period is derived from

$$I = k\Delta GP$$

where k is an average capital coefficient computed with respect of the period as a whole. Capital coefficient is functionally related to investment. After a certain point, further increase in investment will increase the value of k. Until the relative increase of k is less than the relative increase of I, the change is worth while, because ΔGP will be expanding. The optimisation rule follows directly:

$$\eta_{kI} = 1$$

expand investment until the elasticity of the average capital coefficient with respect to investment becomes equal to one.

8. The choice we shall have to make may be clarified by recalling our *IP* diagram.

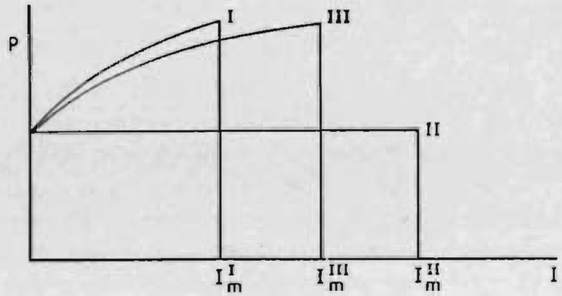

FIG. 2.—The Choice of an Optimum *IP* Function

Again, as in Fig. 1, the areas under the curves represent total additions to social product in a period of so many years. All alternatives are considered with respect to the same period of time. In this period various amounts of total investment may be undertaken and, given the total amount, various combinations of successive yearly investment doses are possible. In fact, the number of conceivable variations in both categories is unlimited. We shall not bother about the latter, and from the former we choose three representative cases. Before we proceed let us clearly realise that we are not analysing the curves themselves but the areas under the curves. Still, in drawing the curves one rule has to be observed: given the length *m* of the period under consideration and given the total amounts of alternative investments in this period, I^{I}_{m}, I^{II}_{m} and I^{III}_{m}, distribute investment over time in such a way as to maximise total potential output increment in every case. In other words, to be comparable with each other, curves should be optimum curves with respect to the chosen total amounts of investment. Now, suppose that our curves satisfy this condition. Then their shapes become irrelevant, and in searching for an optimum optimorum the relative areas have to be compared. The three representative areas in the diagram are chosen so as to represent the following relation between respective additions to social product: $\Delta GP^{I} < \Delta GP^{II} < \Delta GP^{III}$.

The first of the three *IP* curves may be conveniently termed the "underplanned solution" (historically the capitalist case); the second one the "overplanned solution" (historically, perhaps, the Soviet case); the third one represents optimum. The first solution shows the lowest rate of growth. The capital productivity (p) is higher than in either of the remaining two

solutions, but the total increment of output is the lowest due to the underinvestment. The second solution achieves much higher rates of growth than the first one, but due to overinvestment, negative outputs have been produced.[1] This becomes clear when the alternative II is compared with the alternative III where smaller total amount of investment creates larger addition to the social product than in II. The area under the curve III is the largest, and therefore this alternative should be chosen in formulating investment decision (granting that the sole aim of economic planning is production maximisation).

9. In paragraph 7 the term "specified period *m*" was used. The best practical approach to determine this period seems to be to identify it with the longest maturation period of the important productive factors. We may then tentatively proceed in the following way. The average maturation period of investment is about three to four years. It will take some four to ten years to achieve any appreciable change in the number of skilled and highly skilled workers. And the training of the university graduates and the development of the research institutions will require more than

[1] There are some reasons to believe that this was what actually happened in the pre-war years of Soviet industrialisation. Apart from the existing verbal descriptions of the economic conditions of that time, the following statistical comparisons may perhaps shed some light. According to the last Soviet statistical publication (*Narodnoe Khozjajstvo S.S.S.R.*, *Moscow*, 1956, pp. 28 and 158–9) in the period 1928–40 the total productive capital and national income were expanding at approximately the same rate (indices: 1928 = 100, 1940 for capital 445 and for income 514). If these figures have any value as indicators, then they indicate that in the whole period of twelve years the production coefficient remained substantially unchanged (as it is assumed in the diagram above, curve II). It would seem plausible to assume that at the beginning of planning in the U.S.S.R. the production coefficient and the share of investment were not substantially higher than those at the beginning of the planning in Yugoslavia. Further, according to the same source, the cumulative annual rate of expansion of total (?) investment (*kapitalnye vloženija*) in the period 1929–40 (no figures for 1928) was 17%. The rate of growth of gross investment (housing, etc., excluded) in Yugoslavia in four adjusted years was 10% and production coefficient increased in the next four-year period from 0·2 to 0·4 (it will be recalled that p computed on the basis of new investment would be more correct). If on the basis of these data Soviet and Yugoslav *IP* curves are constructed, their shapes turn out to be very similar to those of curve II and curve III in the diagram. The whole comparison is obviously very hypothetical. But it seems to suggest a possibility of an, at least occasional, overinvestment practice in the U.S.S.R. in the first phase of industrialisation, and it also shows the possibility of achieving quite conclusive results on this matter if statistical series were more reliable and more comparable with each other.

It may also be noted that only in a situation of overinvestment, when the increments of output become negative, the popular belief that high level of investment is incompatible with the high level of consumption is a correct description of facts. Overinvestment cuts consumption at both ends: the total product is smaller than it otherwise would be, and in this smaller product the share of investment is greater than necessary. Does not this explain another well-known feature of the Soviet economic development, namely the relatively slow expansion of personal consumption, what hasty critics of the system identified with *every* planning for rapid growth? Particularly if one bears in mind that the already diminished total volume of consumption was further diminished by relatively heavy defence expenditures and large communal consumption in general, so that not much was left to personal consumption. Consider Yugoslav example again. Instead of doubling production coefficient, it would (hypothetically) have been kept constant and the amount of investment could have been doubled (increasing its share to more than 40% of *GSP*). In this hypothetical case all additional investment would have meant equal loss of consumption without any gain in the higher rate of growth.

ten years. Perhaps twelve years or three four-year plans will constitute the planners' horizon.[1] Then within this period the expansion of A and I will have to be adjusted in an optimum way.

The maximisation over the whole period may mean that there will be no maximum at any of the three sub-periods. Productive factors are always to a certain degree flexible. The students may be sent to factories instead of to universities and the output of this year will be increased. Or we may plan the development of an industry with the short maturation period but with a bad production coefficient; the output in the sub-period will again be increased. Therefore optimum and maximum in the sub-periods have to be carefully distinguished.

This also calls for a careful handling of the concept of marginal efficiency of investment (*mei*). To achieve maximum output *mei* has to be made zero. But in any sub-period and for any *individual* business *mei* may be greater than zero. This explains the secret of the positive rate of interest in a situation of a zero or negative *mei*. Interest rate is the price for the scarce factor of production called capital, when capital is being distributed among alternative uses. But interest rate has nothing to do with the marginal productivity of the *social* capital (entire capital treated as a whole). If investment is pushed beyond the frontier of A, the additional factory, being a modern one, may very well earn substantial profit. But at the same time the process described by the curve II in Fig. 2 will take place: external diseconomies *with respect to the economy as a whole* will outweigh the positive contribution of the *additional factory*. Additional investment simply reduces the general efficiency of the economy and, for that matter, of the capital employed. This is a reduction in total, not only in marginal product, by an absolute amount, not only of the last unit of investment relatively to the last but one. The distinction is similar—but not more than similar—to that between marginal product and marginal revenue in the theory of the firm: marginal value product may be greater than marginal cost, and still the firm will experience a decrease in profits because marginal revenue is less than marginal cost. Long before an additional factory is earning zero returns additional investment will cease to be productive. Thus one may conceivably speak of two *mei*: one with respect to the last unit only, and another one with respect to the economy as a whole. The latter determines the optimum rate of investment. In this respect the empirical p curves should also be handled with caution. Reconsider our diagrams.

[1] In fact, planners' horizon may be determined quite exactly (theoretically). The principle is essentially the same as in engineering design. Economic mechanism is not an absolutely precise machine; on the contrary, the adjustment tolerance of its cogs is quite considerable. On the other hand, additional gains in the precision of planning are rapidly decreasing with the extension of the horizon. The point where these "precision gains" become equal to the existing "tolerance margins" of the economy represents the limit of the desirable length of the planning period. It should be borne in mind that a long-period plan is not at all absolutely binding. Every year the horizon is extended for another year and the plan will be adjusted to new circumstances.

In every successive investment period the amount invested may increase, increasing at the same time the value of p (see Fig. 1). As p is rising throughout the period one is tempted to say that marginal efficiency of investment is rising (the shares of other factors being neglected for the moment). But taking the period as a whole, the optimum IP curve may lie below or above the actual curve (see Fig. 2). In the first case (curve I) *mei* is actually positive, and all potentialities of the system have not been exhausted. In the second case (curve II), although marginal $p = 0$, the true value of *mei* is negative: overinvestment has occurred. An analytically meaningful definition of *mei* will then read as follows: marginal efficiency of investment is the (positive or negative) change of (actual or potential) total increment of output caused by an additional unit of investment anywhere in the maximisation period. In other words, *mei* has meaning only with respect to the whole maximisation period.

It should also be noted that the concept of *mei* is used here in its gross formulation; rewards for factors other than capital have not been deducted. Thus $mei = \frac{d\Delta GP}{dI}$, it is a change of physical output per unit of additional investment. This is the consequence of our basic assumption of the absorptive capacity of an economy as being measurable by the volume of the total increment of physical output the economy is capable of achieving in a specified period. As a moment of reflection will show, in this conceptual framework there is no need for a net concept.

The Nature of Period Maximisation

10. There is a difficulty, not so far mentioned, in the procedure of period maximisation. Because of the interdependence of factors and the time lags the maximisation has to be carried out with respect to infinity. There is logically no possibility of confining it to a single period. We may, for instance, plan the output of university graduates to-day for the beginning of the twelfth year. But what about all the intervening years? If, say, in the eleventh year we again reduce the expenditures on the university training (and send students to factories [1]), the output of the first twelve years may be overmaximised, but it will certainly not be maximised in the period of next twelve years and in the whole of the two periods combined.

It must be possible to design a rule which will prevent overmaximisation of the kind just described. Proceed in the following way. Construct first IP curve on the strict maximisation principle. Estimate the difference of the potential future productivity of factors when expanded as compared with the state in the first IP curve. Then readjust A factors and redraw the

[1] This, naturally, does not imply regimentation, but refers to those marginal changes which may be achieved by material stimulation or destimulation or, in this case, by changing the standards of admission. Besides, these changes—being marginal—are probably practically not very important, and the whole problem is much more a problem of theoretical principles than of practical application.

IP curve. If the resulting loss of output in this maximisation period is less than the next period's increment due to the present readjustment, the change is worth while making. Now, it is obvious that the productivity in the next period cannot be estimated with precision. To achieve a precise estimation one would have to draw the *IP* curve for the next period as well, and to do this one more *IP* curve for another period ahead and so forth *ad infinitum*. As we cannot predict infinity, we have apparently encountered an insurmountable difficulty.

Yet, is this difficulty really different from that inherent in any prediction? Can we say we *know* what is going to happen to-morrow, or even next hour?—As soon as we leave the static world, we are in the world of approximations, all differences in precision are only differences in degree. We choose the length of our maximisation period not because we pretend to *know* the pertaining *IP* curve, but because this procedure is operationally simpler and yields better *approximations* than if we deal with a much shorter or with a much longer period. In the same way we do not *know* the *IP* curve of the next period, but we have some elements to estimate the *approximate* productivity of factors which are of interest for us. The further we go into the future the less certain our approximations are—but also the less relevant for our present position.

The difficulty encountered is, therefore, not peculiar to this particular approach. It is the "aporia" of the real world, and may be escaped only in a static world with no changes. And if we prefer not to follow the recipe of ancient Eleatic philosophers and eliminate movement as logically wrong—we are left to accept things as they are and to conceive planning as an infinite process of gradual approximation. The task of the economist is to design methods which will increase the degree of approximation in a dynamic world—not to remove change from it.

The Evaluation of Present Sacrifices

11. So far, well and good. The examination has in fact been predominantly theoretical. But—as it is often said, and may well be said once again—it is hardly a practicable proposition to push investment so far as to make $mei = 0$. This would imply an exceptionally great increase of investment, which then involves considerations of present sacrifices of consumption. If it were for only 1 or 2% of social product to be added annually to already existing investment funds, the whole thing might have practical value. Otherwise, who can make consumers agree to refrain from consumption for the sake of future generations?

Here we may pause for a moment. It is exactly those 1 or 2% which are needed. To demonstrate this a simple model will be constructed. The model will be as crude as it is simple, but it will give us an idea of the order of values, and this is all we need.

Suppose there is a regularly growing economy with the share of investment in social product $s = 15\%$, capital coefficient $k = 3$ and, therefore, with the rate of growth $r = 5\%$. Suppose next that each year respectively 1 and 2% are added to investment fund causing the following year an additional increase in output and increasing thus continuously the rate of growth of output; k is supposed to remain unaffected by these changes. On these assumptions output and consumption will be changing in the following way:

Year.	Pattern I.			Pattern II.			Pattern III.		
	Output.	s, %	Consumption.	Output.	s, %.	Consumption.	Output.	s, %.	Consumption.
0	100	15	85	100	15	85	100	15	85
1	105	15	89	105	16	88	105	17	87
2	110	15	94	111	17	92	111	19	90
3	116	15	98	117	18	96	118	21	93
4	122	15	103	124	19	100	126	23	97
5	128	15	108	132	20	105	136	25	102
6	134	15	114	140	21	111	147	27	108
7	141	15	120	150	22	117	161	29	114
8	148	15	126	161	23	124	176	31	121
9	155	15	132	173	24	132	194	33	130
10	162	15	138	187	25	140	216	35	140

In the first year the change in investment policy *relatively* reduces consumption; since then the rate of growth of consumption is constantly increasing. It will take respectively nine and ten years to surpass the level of consumption of the original pattern of growth. The result is not substantially altered if the share of investment or the capital coefficient is changed (see Appendix I).

The differences in the consumption during the whole period are relatively small; the greatest, in the middle of the period, is of the rank of 6% which indicates the lag of only one year. The crop fluctuations in underdeveloped countries, the industrial fluctuations in developed countries and, in both of them, the unproductive expenditures of government such as defence, not to speak of war expenditures, are all far in excess of this difference. Bearing this in mind, it is realistic to expect that if a nation is asked to accept a programme which envisages that a certain level of consumption will be achieved in the fifth instead of the fourth year, with all the good consequences [1] afterwards—it will wholeheartedly accept this programme,

[1] One of the good consequences may appear as a considerable "free gift of nature" caused by relativity properties of the economic system in motion. Namely, capital costs, *ceteris paribus*, are not fixed, but are conditioned by the speed of growth. The greater the speed of growth the smaller capital costs per unit of output will be. Or, alternatively, the same piece of equipment will, *ceteris paribus*, produce about 3½ times larger total output in an economy growing at the rate of 10% than in the one growing at the rate of only 3% (if productive life of equipment is 30 years and the productive capacity of it remains approximately constant); for the rigorous demonstration see my "Depreciation Multiplier" in the forthcoming issue of the *Manchester School*.

and by accepting the programme it will push its economy on the path of the maximum rate of growth, for this programme implies an annual increase of the investment at the rate between 12% (pattern II) and 18% (pattern III). And exactly in this range the maximum rate of expansion of *A* is likely to lie, as it has been demonstrated by Soviet and Yugoslav economies which occasionally even overshot the mark of productive investment.[1]

The general result of the analysis so far consists, then, in the conclusion that the production maximisation is not only a theoretically determined but also a practicable proposition. The final question which remains to be answered is: where is maximisation of production likely to lead? This brings us to the fascinating subject of the limits of the economic science, to which the last few paragraphs are dedicated.

"Underdeveloped" and "Developed" Economies

12. The maximisation discussion has hitherto implied the case of an underdeveloped economy. This assumption has now to be stated explicitly by defining the underdeveloped economy as one where the share of investment is less than optimum and/or the level of *A* factors is low (as compared with already achieved standards in other countries). In this sense all existing economies are underdeveloped. And this will not be surprising if one takes into account the transitional state of to-day's world economy.

As the economy expands, the level of *A* factors will rise, and this will change their productive functions. Once poverty economics is left behind, and people are well fed, have reasonable leisure time and enjoy a healthy life, factors *G* and *H* lose their place in *IP* function. They are no longer productive agents but only ends in themselves.

Further, it seems reasonable to assume that after a while every socio-economic system in its development reaches a stage of relative stability and that this applies to the planned economy as well. At this stage economic

[1] The Soviet case was already discussed in an earlier note. The Yugoslav case refers to the period of the first Five Year Plan, when, with the then given state of *A*-factors, as some case studies tend to suggest, too large an investment programme has been carried out to be productively absorbed. Gross investment (housing, etc., excluded) in the period 1947–52 was expanding at the rate of 15% per annum. The share of gross investment in gross product surpassed already in 1951 the mark of 30%, and since then not only the share but even not the absolute amount of annual investment has been achieved again. The labour and capital productivity were low, even declining. Since the end of the first F.Y.P. (1952) the tempo of investing has been gradually slowed down, *A* factors have undergone continual readjustment and the rate of growth of output has, if anything, increased. It may be added that in spite of the restricted expansion of investment since 1952, even to-day, as the annual conference of the Federal Chamber of Industry established in May 1957, manufacturing and mining have less than 80% of the necessary number of skilled workers and only 50% of highly skilled workers, the training of new skilled workers is still lagging behind the *current* needs, and the university output of graduate technicians is only slightly more than one-half of the necessary number (*Borba*, May 7, 1957), although—to illustrate the last statement with two comparisons—in relation to the population there are already twice as many university students in Yugoslavia than, *e.g.*, in Great Britain, and by 1955 the number of technology students had been increased five times as compared with 1939.

and political organisation will be more or less stabilised, all major potential innovations in this sphere exhausted, and the system will perpetuate itself almost automatically. This the factor *O* may be dropped out too.

In this way *A* will be reduced to *Kn*, which remains the only limitational factor of growth. The "intellectual capacity" of a community will provide unsurpassable limits for the productive application of investment and so for the speed of expansion of its economy.

13. The last statement may cause some difficulties. For it is obvious that potential knowledge is unlimited and that, therefore, expenditures on research, however large, must be productive. Therefore *Kn* expenditures would absorb the whole social product, and thus the criterion for the distribution breaks down.

However, there is a fallacy in this reasoning, and it is of exactly the same nature as the apparent contradiction of simultaneously positive and negative *mei* analysed in paragraph 9. It resulted here from the confusion of the ultimate result with the speed of change Skill and knowledge will always be increasing, but the physiological substratum and social habits impose quite definite limits to the *speed* of the change. The increase of production requires continual readaptation of the whole social structure. This may not be evident in a slowly expanding economy. Yet, suppose that the rate of growth is 10%. Then in a generation of two twelve-year periods output would increase 10 times. Our children would have to manipulate an output 100 times as great, and our grandchildren an output 1,000 times as great. An underdeveloped and poor country of to-day would after only seventy years manipulate an annual social income of some $100,000 *per head of population*—do not these figures sound startling? Obviously, there is a physical limit to the rate of expansion to which society is able to adapt itself.

14. The process of development or transition may be advantageously described diagrammatically in the following way:

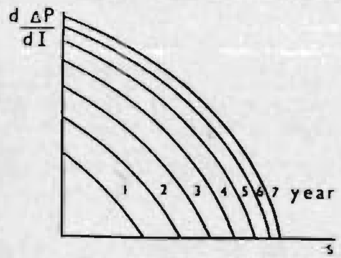

Fig. 3

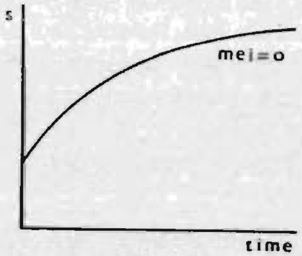

Fig. 4

The Transition from a Low- to a High-investment Economy

The left-hand diagram indicates the increase of the share of productive investment in every successive year (denoted as years 1 to 7). The continual increase of *s* pushes *mei* curve every year to the right. These impulses tend to decrease. If then all $mei = 0$ points are plotted on an s–time diagram the resulting curve will increase at a diminishing rate (the right-hand diagram). The rapidly growing first section of the curve (which may have also an S-shaped beginning) shows the transition from the low-investment and unplanned to the high-investment and planned economy. After all economies of adjustment of I and various A factors are exhausted and Kn is left as the only source of the further expansion of A the slope of s curve will tend to flatten. And as s can never reach 100%, the further rise of s curve may proceed only at a decreasing rate, *i.e.*, the curve will approach asymptotically a fixed limit in the infinity. In this second stage, after the new level of a high-investment economy has been reached, the share of investment may for all practical purposes be taken as constant.

Here we reach the point where welfare considerations may be profitably introduced into the discussion.

A New Welfare Theory

15. The problem of economic welfare may be approached in basically two different ways: either starting from the assumption of an isolated individual maximising his economic welfare in a static world or from the assumption of the society (or, if this is a better term, the social individual, *economic zoon politikon*) maximising economic welfare in a dynamic world. Theoretical identity of these pairs of concepts, individualist-static and socialist-dynamic, should have been made clear by the implications of the discussion so far. Here it will be derived explicitly with the purpose of showing that the theoretical apparatus developed in this paper makes it possible to construct an ethically neutral criterion for judging the economic desirability of various types of economic systems.

16. The first approach is sometimes stated in the following categorical form: the absolute limit of maximisation of economic welfare of present generation will be reached when the whole social capital is used during this generation's life-time and nothing is left to the next generation; there exists no economic criterion for judging whether and how much of social capital to leave to the next generation. Instead of " generation " the term " individual " may apparently be used without altering the sense of the contention.

Let us examine briefly the implications of this approach.

Suppose the whole social capital inherited from past generations has been used up during the life-time of the present generation. This would mean an immediate increase of consumption equal to previous share of investment in social product, say by 20% of *GSP*. Yet in a rapidly growing economy this increase of consumption will be reached in four or five years anyway, and afterwards it will be greatly surpassed. Thus, from

the point of view of every individual who expects to live longer than five years it would be very foolish indeed to stop investing.

Further, what does the term " generation " mean? Five years of a child who dies in his fifth year or hundred years of a Methuselah? Presumably everybody is expected to maximise his own welfare. But this is possible only within the consumption possibilities provided by the social productive effort manifesting itself as a single integrated force. At the back of the confusion of the relative with the absolute sovereignty of the *homo economicus* lies the naïve preconception of an individual independent of his social environment. In the modern American as well as in the modern Indian society individuals are free to save or to spend their incomes and shape their consumption functions throughout their life-times in whichever way they find fit. The only choice they *cannot* make as independent individual is in fact the *much more basic choice of smaller or larger total income in their life-times*; whatever the Indian does, the income of his American twin is ten or more times higher. In this respect their choice is determined by the society in which they live: by the level and the speed of development of the social productive forces.

Thus individualist approach breaks down completely and we are left to try the second alternative.

17. Granting that individuals are free to choose their occupations and to shape their consumption functions in time, everything outside this decision sphere depends upon social decisions. In the first decision sphere the individual decision-maker may be absolutely independent, in the second he cannot be; this is simply one of the natural properties of the economic system as such, and no amount of economic theorising can alter it. Production and distribution are social processes irreducible to independent individual economic decisions either in the privately or in the socially owned economy. In the former the production function (and thereby the volume of available consumption) depends on more or less haphazard market forces which are likely to lead to boom as well as to slump regardless of our individual intentions. In the latter it is more or less planned with the purpose of achieving certain socially determined aims. The maximum rate of growth of output depends on the expansion of the production capacity of the economy as a whole. Now, if the share of investment is constant and production is always maximised, consumption will be maximised also. And exactly this was the result we wanted to achieve.

In fact, the restrictive assumption of the constant s is not necessary. If the unmanageable concept of " generation " is eliminated, a meaningful basis for welfare maximisation will be provided by the following proposition: Maximisation of welfare of *every* individual means maximisation of the total volume of consumption in the life-time of *any* individual consistent with the similar maximisation of *any other* individual. And this requirement is automatically satisfied once the economy has been pushed on to the path of maximum growth. For any other policy would be equivalent to the diminution of welfare for some individuals without any increase in welfare for anybody else (see Appendix II).

Conclusions

18. The findings of the analysis may now be summed up. The principle of output maximisation has provided a key for a determinate and practicable solution of the problem of optimum investment and has also provided a basis for an operationally meaningful welfare maximisation in a rationally planned economy. As the economy expands the basic investment decision will soon be reduced to the adjustment of investment to the human capacity to produce inventions and innovations. This will determine the physical upper limit for the rate of growth and will probably cause a relative stabilisation of the share of investment. Judging from the experience so far, this stabilisation for the foreseeable future may be expected to occur at around 30% (possibly more) of productive investment in social product, which will sustain a rate of growth of output of 10% or more per annum.[1] This doubling or trebling of the rate of increase of the population's well-being is what we may expect from the planned economy as compared with economies we used to know. This will mean the exhaustion of the purely economic possibilities of growth, and as such represents the limit to what the economist has to say on this subject.

BRANKO HORVAT

Manchester.

Appendix I

Characteristics of the Model

All important characteristics of the model may easily be derived from the rates of growth of investment and consumption once they are expressed as algebraic functions.

Denote capital coefficient, *i.e.*, the ratio between investment in this year and the resulting increment in output next year as k, investment as I, consumption as C, output as P, rate of growth of investment as r, rate of growth of consumption

[1] It may be worth noting how the naïve though widespread belief, that the rate of growth is bound to decline in a mature economy, proved to be wrong, as the economic history of at least one country, the U.S.S.R., has demonstrated. In the period of the last eighty-five years Soviet economy has experienced the following sequence of the rates of growth of national income: 1870–1900, 3½%; 1885–1913, 4½%; 1928–37, 7%; 1948–55, 9% (M. C. Kaser, "Estimating the Soviet National Income," ECONOMIC JOURNAL, 1957, p. 101). The post-war Japanese and German economies have displayed similarly high rates of growth. And in the last eighteen years American economy is expanding at a rate far higher than the past eighty-year average. If the reasoning in the text is correct, the decreasing *rise* of the rate of growth, and not the diminution of the rate of growth itself, is what one may expect in a developed economy. All this obviously on the assumption of no obstacles on the part of the institutional set-up. A separate problem arises from the use of "borrowed technology" in the early stage of the economic development. Once foreign credit in technological knowledge is basically exhausted, this may depressively influence the rate of growth. However, no such phenomenon seems observable in the Soviet case.

as ρ and the share of investment in product as s. Investment in two successive years is given by

$$I_{t-1} = s_{t-1}P_{t-1}$$

$$I_t = s_t P_{t-1}\left(1 + \frac{s_{t-1}}{k}\right)$$

and the rate of growth in any one year t by

$$r_t = \frac{I_t}{I_{t-1}} - 1 = \frac{s_t}{s_{t-1}} + \frac{s_t}{k} - 1 \quad . \quad . \quad . \quad . \quad (1)$$

If every year, 1 and 2% respectively are added to s (1) may be expressed in two alternative ways:

$$r_t = \frac{0{\cdot}01}{s_{t-1}} + \frac{s_t}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.1)$$

$$r_t = \frac{0{\cdot}02}{s_{t-1}} + \frac{s_t}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.2)$$

It is evident that the rate of growth of investment changes through time. It will be declining when the following inequality holds good

$$\frac{1}{s_{t-1}} - \frac{1}{s_t} > \frac{1}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The inequality holds good up to $s = 17\%$ for 1% additions and up to $s = 24\%$ for 2% additions ($k = 3$). The deviations from an average rate of growth are relatively small.

Similarly, consumption in two successive years will be given by

$$C_{t-1} = P_{t-1}(1 - s_{t-1})$$

$$C_t = P_{t-1}\left(1 + \frac{s_{t-1}}{k}\right) - s_t P_{t-1}\left(1 + \frac{s_{t-1}}{k}\right)$$

Therefore

$$\rho_t = \frac{C_t}{C_{t-1}} - 1 = \frac{(1 - s_t)(k + s_{t-1})}{k(1 - s_{t-1})} - 1 \quad . \quad . \quad . \quad (3)$$

The *volume* of consumption will be normally increasing. The condition for this is that the increment of product is not exhausted by the increment of investment

$$\Delta P_t > \Delta I_t$$

$$\Delta P_t = \frac{s_{t-1}}{k} P_{t-1}$$

$$\Delta I_t = s_t P_{t-1}\left(1 + \frac{s_{t-1}}{k}\right) - s_{t-1}P_{t-1}$$

$$s_{t-1}(1 - s_t + k) > s_t k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If annual addition to s is respectively 1 and 2% (4) may be expressed as

$$s_{t-1}(0{\cdot}99 - s_{t-1}) > 0{\cdot}01k \quad . \quad . \quad . \quad . \quad . \quad (4.1)$$

$$s_{t-1}(0{\cdot}98 - s_{t-1}) > 0{\cdot}02k \quad . \quad . \quad . \quad . \quad . \quad (4.2)$$

or, as an approximation

$$100\, s_{t-1} > k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.1.1)$$

$$100\, s_{t-1} > 2k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.2.2)$$

This means that, if $k = 3$, starting share of investment must be greater than 3% in the first case and more than 6% in the second case to prevent consumption falling absolutely in the next year.

The *rate of growth* of consumption is constantly increasing for all feasible values of s and k as may easily be found by putting $\rho_{t+1} > \rho_t$.

Realistic changes in the assumptions of the model will only slightly change the results described in the text. Thus the smaller the value of capital coefficient k, the greater, *ceteris paribus*, both rates of growth will be. Further, the smaller the k, the smaller the original share of investment—and the smaller annual additions to it—the sooner the level of consumption of the steady growth pattern I will be reached. If k decreases from 3 to 2, or if the starting share of investment is $s = 9\%$ instead of $s = 15\%$, or if annual additions to investment are 1% instead of 2%, the level of the consumption of the pattern I will generally be reached one to two years earlier.

Appendix II

Equivalence Theorem

In paragraph 17 it was contended that any movement off the path of maximum growth is equivalent to the diminution of welfare for some individuals without any increase in welfare for anybody else. Here a formal (logical) proof for the contention will be supplied.

The decision to push the economy on to the path of maximum growth is a once-for-all decision. Therefore generalisations about it, in the traditional manner, do not seem to have much sense. Probably nearest to common sense would be to ask the referendum-question as it was formulated in paragraph 11. But even such a question is still much too formal to have a real value for the economic theory and policy. This is so because the affirmative answer implies not simply a choice with respect to economic welfare but also a choice with respect to different socio-economic institutions; to achieve maximum growth the economy has to be planned. However, once this basic decision has been made and the economy starts moving along the path of maximum growth—and this has been the assumption of this paper—formal generalisations about economic choices have sense and, moreover, they lead to some interesting new results as compared with the traditional welfare theory.

The choice we face in such an economy is formally the same as before: either to invest more or to invest less as compared with the original investment pattern. The first alternative is dismissed at once because it leads to negative increments in product, with a consequence that everybody loses and nobody gains. The second alternative implies that *IP* has been shifted to maximise consumption in a particular sub-period. In this case some individuals would gain at the expense of others and at the expense of their own future consumption. In physical terms this gain is always incomparably smaller than the resulting loss, because the gain, being temporary, is finite, and the loss, being permanent and, moreover, expanding, is infinite. The problem is a little more complex in utility terms.

Traditional approach would imply that saving has to be pushed to the point where marginal disutility of saving becomes equal to the marginal utility of investment. In the economy on the maximum growth path marginal productivity of investment (*mei*) is zero. With *mei* = 0 marginal utility of investment (*mu*) is zero too. If this is so, marginal disutility of saving (*md*) must also be zero at this point. If it were positive, it would be necessary to reduce investment to make *mei* > 0 and *mu* = *md*. This conclusion applies to every moment of time. Therefore it would never be possible to achieve maximum rate of growth. But this would contradict our starting assumption.

Thus the first choice of maximum growth implies that marginal disutility of saving is zero. If in a growing economy *md* = 0 at one time it has to be zero for ever, because increasing consumption implies decreasing marginal disutility of saving and, by definition, *md* cannot diminish below zero (and even if it could, this would not change the final conclusion). Therefore, if maximum growth alternative is once chosen, it will continue to be chosen. Every other policy—the restriction or the expansion of investment as compared with the optimum rate—brings about losses for some individuals without any gain for anyone else.

If the foregoing reasoning is correct, the theorem will have two important implications. First, in a situation of maximum growth traditional utility considerations become irrelevant. And, second, once the first choice of optimum *IP* is made, traditional utility logic becomes invalid: disutility of saving does not change (continuously) with the welfare of the community and therefore *md* = 0, and cannot decrease with increasing consumption. These results, being mere logic, are perfectly general. If in some countries—for some *historical* reasons or others—the decisive first choice has not yet been made, this cannot change the *logical* structure of the set of concepts used. It suffices to accept that the initial decisive choice is feasible and all consequences follow at once as generally valid. Contrary to traditional assumptions, marginal disutility of saving is *not always* positive and is *not always* decreasing with increasing consumption. If there is no regularity, no *a priori* theory can be developed and all statements become tautologies: the choice has been made because it has been preferred. If there is a regularity, then *md* = 0 is a universal characteristic of utility function, and as such renders utility considerations irrelevant. In fact, what appears to be wrong is not the concept of utility as such but the mode of its marginalist application in the field of macroeconomic investment choices.

Finally, it is worth noting how, as a by-product of this analysis, the well known von Mises–von Hayek proposition about rational economic choice in two alternative economic systems will have to be reversed: consistent investment choices appear now to be impossible in the unplanned economy. If it is chosen to achieve maximum growth of consumption, planning is indispensable by definition. If the choice is formulated in the traditional manner a certain regularity in investment will have to be observed; in fact, the share of investment will have to be continuously increasing. The reasons are the following ones. Increasing consumption implies decreasing marginal disutility of saving, this is reinforced if there is an egalitarian tendency in the distribution of income, and decreasing capital coefficient implies increasing marginal utility of investment (*ceteris paribus*). Now, increasing consumption, egalitarian tendencies and decreasing capital coefficient have been observed, but no appropriate regularity in investment. On the contrary, it fluctuated widely and irregularly. Thus if we reject the obviously absurd assumption that investment fluctuations reflect changes in tastes, we are left to conclude that *rational* choice with respect to the amount of investment is possible only in a planned economy. And this choice is the basic economic choice because it determines the rate of increase of consumption.

JUGOSLAV INSTITUTE OF ECONOMIC RESEARCH
Jugoslavenski institut za ekonomska istraživanja
Beograd, Kneza Miloša broj 20, tel. 27-008

REPRINT SERIES

1. — V. Tričković, „Ispitivanje strukture i elastičnosti tražnje", *E k o n o m i s t*, 3-4/1957, 52-88.

2. — B. Horvat, "The Depreciation Multiplier and a Generalised Theory of Fixed Capital Costs", *T h e M a n c h e s t e r S c h o o l*, May 1958, 136-59.